FINAL TECHNICAL REPORT

Title: Autoregressive Representation for Harmonizable Process

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Principal Investigator: Dr. Abol G. Miamee

Institution: Department of Mathematics
Hampton University
Hampton va, 23668

NASA Technical Officer: Dr. Jay C. Hardin

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Purpose of the Project. The main purpose of this research project was to study harmonizable stochastic processes and to develop some type of autoregressive representation which will lead to a useful prediction theory for these processes. The class of harmonizable processes which include stationary stochastic processes is rich enough to contain some interesting classes of nonstationary processes such as periodically correlated processes and V-bounded processes.

Accomplishments. There are at least 10 publications resulted from the support of NASA through this Grant NAG-1-768, where this support has been gladly acknowledged. Now we briefly explained these accomplishments. One of the most important facts known about harmonizable processes is the result which says that every harmonizable process has a stationary dilation. This fact, which has been studied in several papers says that for every harmonizable stochastic process $\{X_n\} \subseteq L^2(\Omega) = H$, there exists a larger Hilbert space $K \supseteq H$ and a stationary process $\{Y_n\} \subseteq K$ such that

$$X_n = PY_n$$

where P is the projection from K onto H. Exploring this basic property of harmonizable process, the Principal Investigator was able to prepare the following two manuscripts which have been submitted for publication:

(a) Recently, existence of stationary dilations for infinite dimensional harmonizable processes taking value in a

Hilbert space has been studied and some interesting results has been established. The Principal Investigator considered the existence of such dilations for those infinite dimensional stochastic processes taking value in a Banach space. This manuscript is called "Spectral dilation of L(B,H)-valued measures and its application to stationary dilation for Banach space valued processes." Copies of this paper which has been appeared in the Indian University J.of Mathematics was enclosed with my December 1990 report to you.

(b) Since the class of harmonizable processes is very large, in order to be able to get good results, one needs to consider subclasses of this class. An interesting question about stationary dilation of a harmonizable process is to go beyond just existence results and try to construct the stationary dilation process in a useful and explicit manner in terms of the original process. In this second manuscript entitled "Periodically correlated processes and their stationary dilation", we actually obtain a nice and explicit format for the stationary dilation process of a periodically correlated process X_n itself. Copies of this paper which has been published in SIAM J. of Applied Mathematics was sent to you with my December 1990 report.

A third manuscript, which has been completed during the period of this grant and the support of NASA Grant NAG-1-768 has been acknowledged, is a paper entitled "On Wiener-Masani

algorithm for finding the generating function of multivariate stochastic processes". This paper which appeared in Annuals of Probability, extends the domain of application of an original algorithm due to Wiener-Masani for finding the predictor of multivariate stationary processes. Copies of this paper was enclosed with my 1990 report.

A forth piece of work which has been completed under the support of this grant was to find algorithms for computing the canonical correlation and the best predictable aspect of future. This work has resulted in a manuscript entitled "Computation of canonical and best predictable aspects of future for the time series" which has been published in Time Series Analysis. Copies of this paper are enclosed with the present report.

The completeness of the spectral domain of stationary stochastic processes is well known. This fact has played an essential role in the development of a nice prediction theory for these processes. However, the completeness question for the harmonizable processes which is definitely equally important has been open for a long time. Recently M.M. Rao announced that the answer to this is positive. After carefully examining these papers and other evidence in the literature. This Principal Investigator (with Prof. H. Salehi) doubted the correctness of this fact. So, we had to find a counter example for the mentioned announced result. A "An example of a harmonizable process whose spectral domain is not complete" is our

contribution to this question and should put to rest any further speculation about this important question. Copies of the preprint of this work which has been published in Scan. J. of Statistics are enclosed with this report.

We (the Principal Investigator and Dr. Jay C. Hardin from NASA Langley) have recently introduced and studied a new class of nonstationary processes which are called Correlation Autoregressive (CAR, in short) which seems very interesting and promising . The study of these processes which started several months ago has now reached a certain stage of completeness for the discrete case. A manuscript entitled "On a class of nonstationary stochastic processes" containing our earlier results on this has been published in Sankhya : Indian J. of Statistics. Copies of this are enclosed with this report. Our investigation on continuous CAR processes resulted in another manuscript which was presented as an Invited Lecture to the 21st Midwestern Mechanics Conference held last year at Michigan Technological University and appeared in its proceeding Developments in Mechanics Vol. 15. The extended version of our results here has been published in J. of Sound and Vib. Reprints of this paper which is entitled "On Correlation Autoregressive processes with applications to helicopter noise" are enclosed with the present report.

Another publication where the support of this NASA Grant is acknowledged is " The inclusion $L^p(\mu) \subset L^q(\nu)$" published in

American Mathematical Monthly. Copies of this paper are also enclosed with this report.

Support of NASA has been acknowledged at several other places including the preface of the enclosed volume on Nonstationary Processes and their Applications which has been edited by the present P.I., as well as the following two papers appeared on pp. 147-164 and pp. 165-175 of the same book:

i) " Periodically correlated processes and their spectrum"

ii) "Correlation autoregressive sequences: A summary"

Some of the results developed on nonstationary stochastic processes were utilized by a Mr. Gregory Smith one of our graduate student, supported by this Grant, to study some helicopter noise data collected by NASA Langley Research Center. These are the subject of his M. Sc. thesis here at Hampton University.